Optimal Groundwater Extraction Under Uncertainty and Spatial Stock Externality

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Motivation

- Small gains from moving from myopic to optimal groundwater extraction
  - Gisser and Sanchez 1980 (.01%)
- Uncertainty
  - Tsur and Tomasi 1991
  - Knapp and Olson 1995 (2.6%)
- Switching of production practices
  - C.S. Kim et al. 1989 – Endogenous crop switching and technology (1-3.7%)

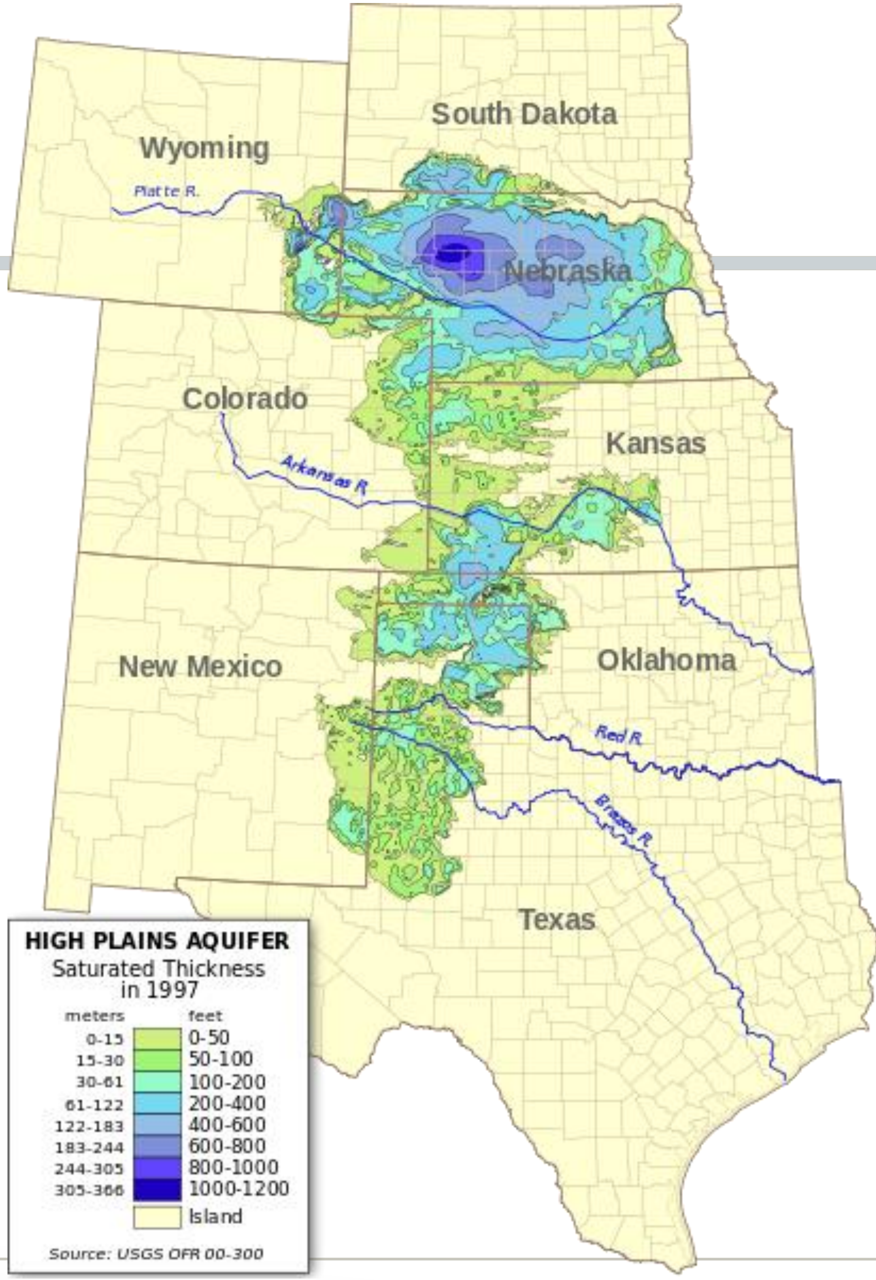
Research Questions

- How much does the loss of irrigated land above an aquifer affect the magnitude of gains from management?

- What effect does uncertainty in rainfall and specification of the stochastic process have on gains from management and optimal policy rules?

Contributions

- Spatial Cone
  - Irrigated area a function of groundwater stock levels
  - Loss of irrigated land, switch to dryland farming practices
- Including climate variability and persistence
  - i.i.d. and Markov chain process
- Gradual stock externality with variable irrigation demand- NW Kansas section of the Ogallala



Background

- 95% of water pumped is for irrigation
- USGS estimates storage of about 2,925,000,000 acre feet in 2011, a 9% decline since 1950
- Saturated thickness varies greatly

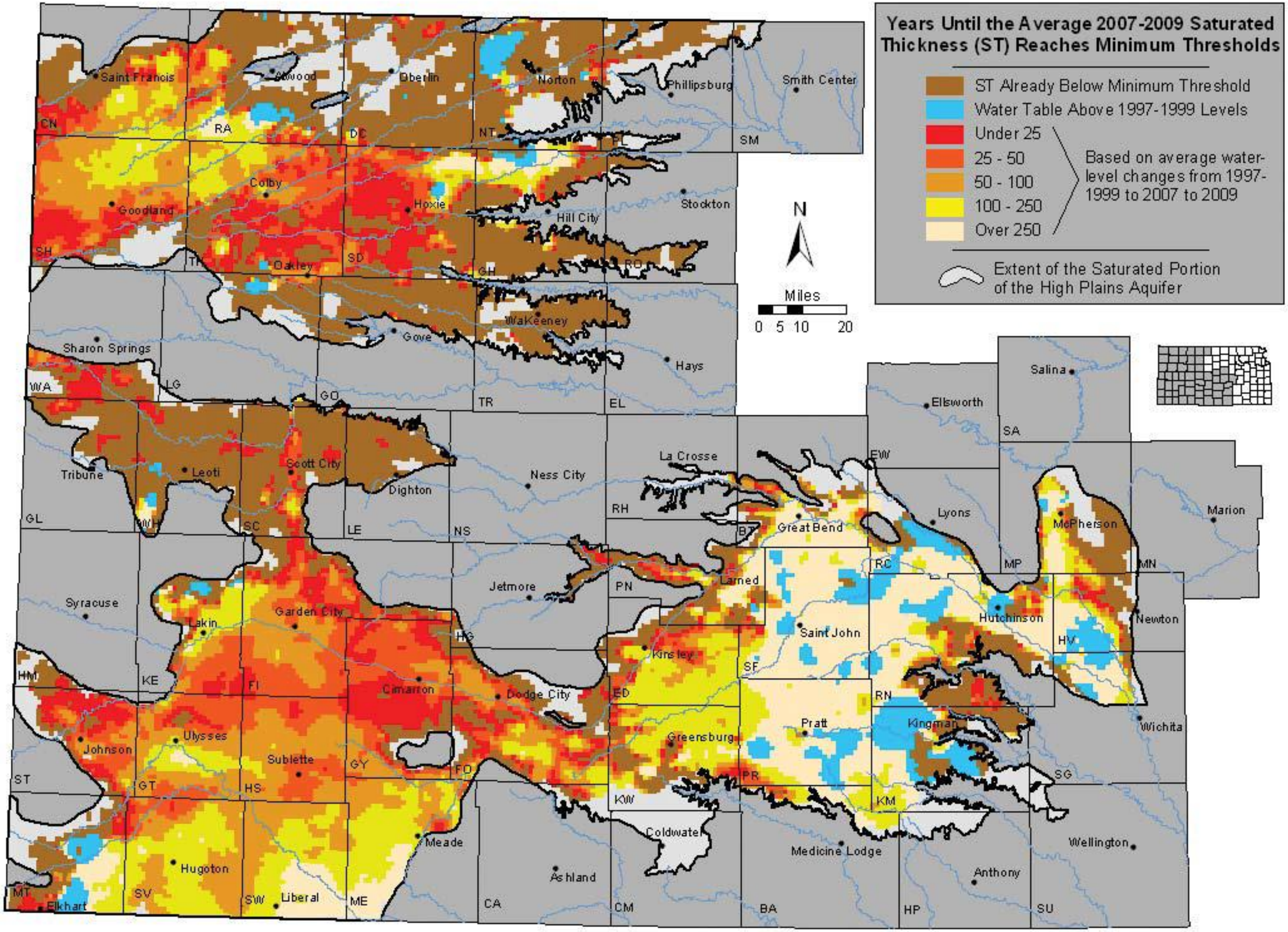
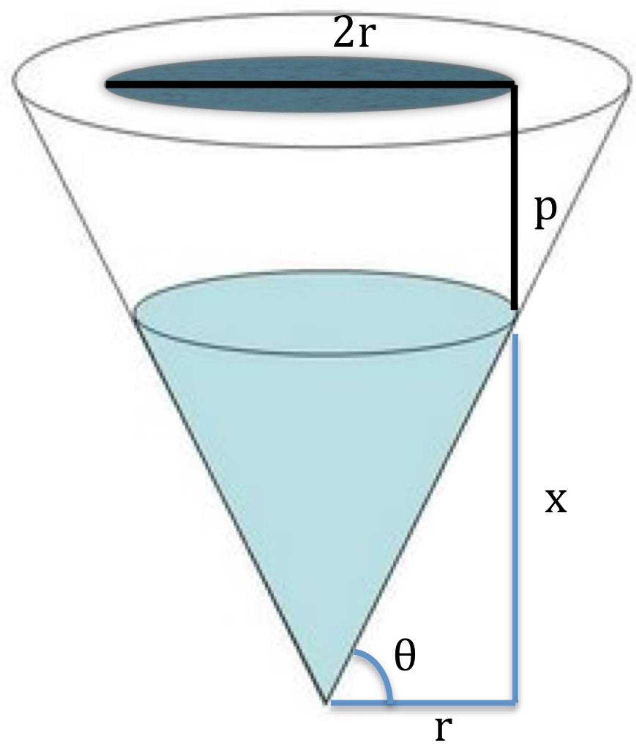


Figure 7—**Estimated usable lifetime (1998–2008) trend for the High Plains aquifer in Kansas. Kansas Geological Survey(2009)**

FIGURE 1
Cone Spatial Model



r- radius of irrigated acreage
p- pumping height
x- groundwater height
θ- slope of depletion

Background

- Common Pool Resource
  - Large area, many farmers, possibly non-binding allocations
- Myopic behavior
  - Make up for lack of rainfall with with irrigation water to maximize one season's profit
  - Little benefit from saving water if others will pump it

  - Sub-optimal by how much?

Model

- Rainfall
  - Deterministic- Average rainfall every year
  - i.i.d. – Random draws from an empirical rainfall distribution
  - Markov chain- Transition probabilities of rainfall states are a function of current year's rainfall (Strikanthan 1999,2001)

Economic Model

- Myopic – Maximize one year's benefit of groundwater pumping
- Optimal – Maximize the present value of the sum of net benefits of groundwater extraction over an infinite time horizon

Economic Model

- Single Year

$$\Pi_t = A[\gamma(x_t) f_I(w_t, r_t, x_t) + (1 - \gamma(x_t))\, f_D(r_t)]$$

Irrigated   Dryland

A- Initial aquifer surface area
w- Groundwater pumped
x- Height of groundwater
r- Rainfall
$F_I(w,r,x)$ – Irrigated profit
$F_I(r)$ – Dryland profit
$\gamma(x)$- % irrigated

Economic Model

- Single Year

$$\Pi_t = A[\gamma(x_t) f_I(w_t, r_t, x_t) + (1 - \gamma(x_t))\, f_D(r_t)]$$

Irrigated — Dryland

- Irrigated Corn, Sorghum on dryland acreage
- Crop yield functions from Kansas State's Crop Yield Predictor

Economic Model

$$\max_{w_t} \sum_{t=0}^{\infty} e^{-\delta t} \Pi_t(w_t, x_t, \gamma_t | r_t)$$

- Aquifer wide profits from groundwater pumping. Infinite time horizon

$$s.t$$

$$\dot{x_t} = \frac{R + (\alpha - 1)W_t}{\gamma_t AS}$$

- Equation of motion

$$\gamma_t = a(x_t)$$

- % irrigated

$$\gamma_0 = 1$$

$$x_0 = \bar{x}$$

$$x_t \in [\, \bar{x} \,, \underline{x} \,)$$

Methods

- Discrete Stochastic Dynamic Programming

- Computationally solve for value function
  - Value function iteration
  - Recover policy function or optimal extraction rules at groundwater heights and rainfall states

- Simulate rules through time with either optimal extraction or myopic behavior

Simulation

- Parameterized for NW Kansas - GWMD 4
  - 3.11 million acres
  - 373,200 acres irrigated
- Simple rainfall states
  - Deterministic- Average
  - Stochastic- High, Average, Low
  - Markov Chain- H,A,L with empirical transition probabilities

TABLE 1
Parameter values for a section the Ogallala Aquifer
Northwest Kansas Groundwater Management District 4

Parameter	Description	Value
C_0	Intercept of pumping cost equation	$104/a-ft
C_1	Cost of pumping	$.11 /a-ft/ft
R	Natural recharge	199,040 a-ft
A	Aquifer area	3.11 million acres
	Initial Irrigated acres	373,200 acres
$\bar{X}$	Land surface	943 ft
$\underline{X}$	Lower aquifer bound	741 ft
S	Storitivity	.17
α	Irrigation water return	20%
x_0	Initial water level	917 ft
β	Discount factor	.96%
r	Rainfall states	
	High	2 ft
	Average	1.58 ft
	Low	1.25 ft

Optimal Policy Functions at Rainfall States (i.i.d.)

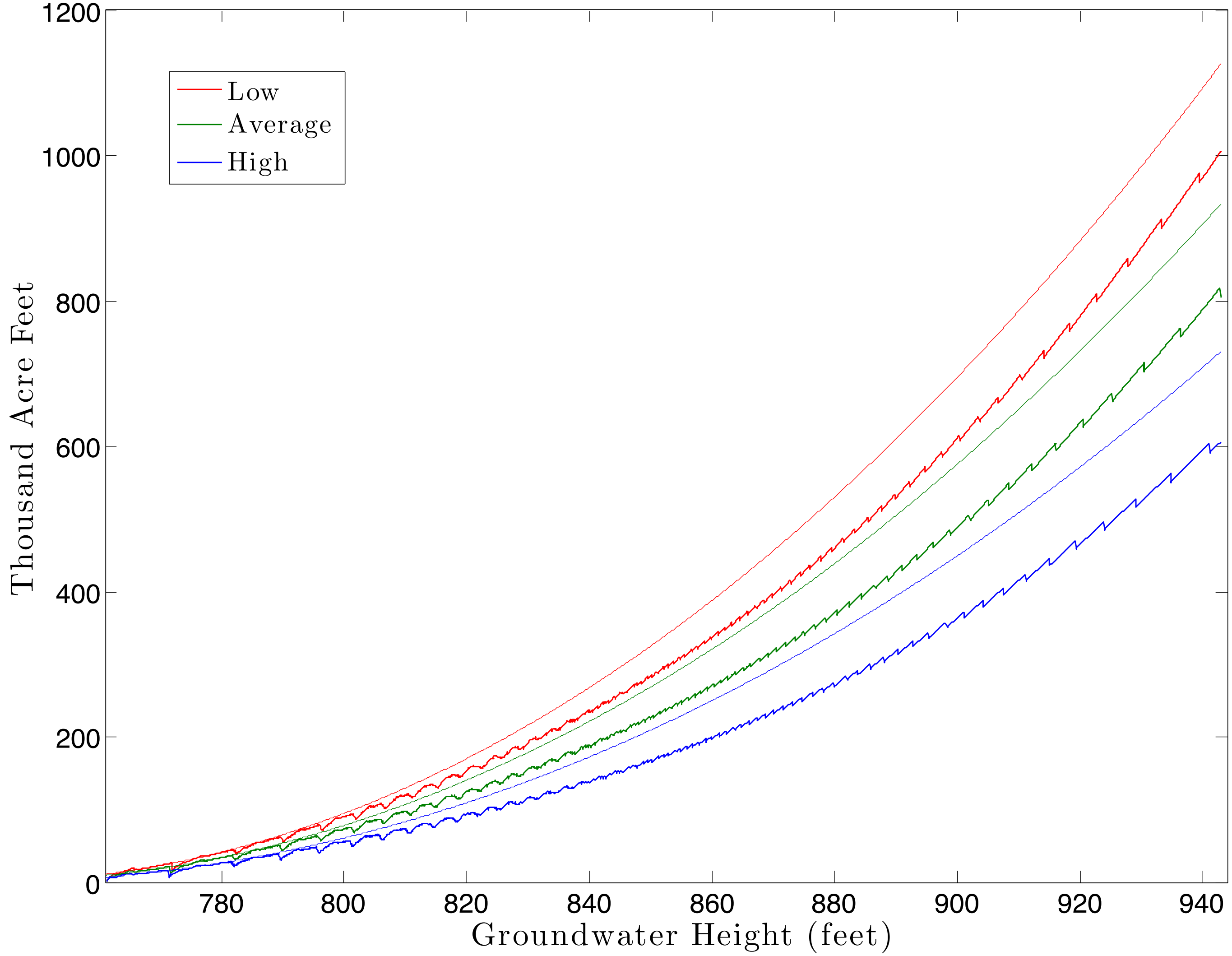


FIGURE 5
Groundwater Height Over Time

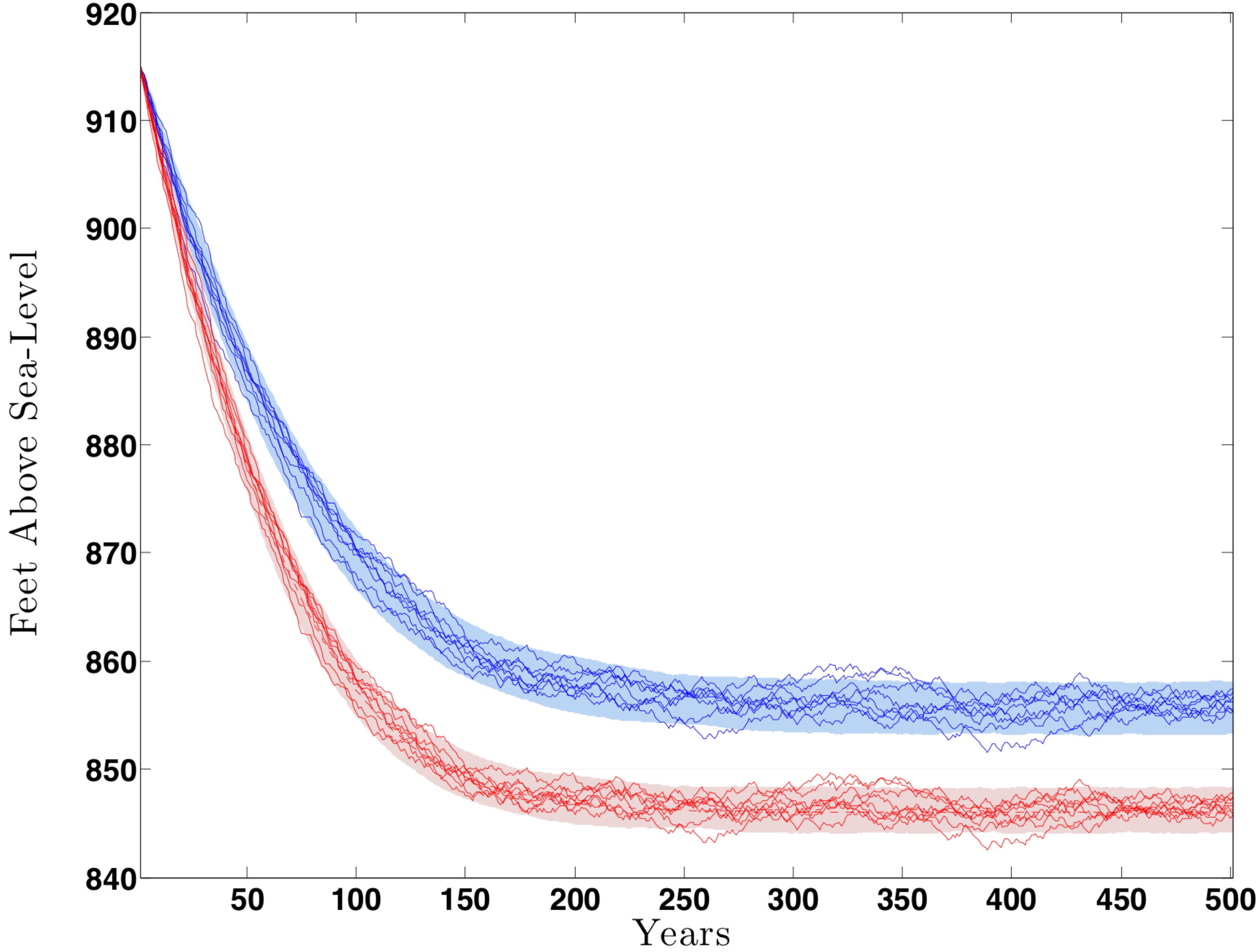


TABLE 3
Welfare Gains From Groundwater Management
Total Discounted Profit (Billion $)

	Perfect Competition	Optimal Policy	Difference	% Gain
Deterministic	$ 8.57	$ 8.68	$ .107	1.24
Stochastic	$ 8.43	$ 8.55	$ .117	1.39
	(.139)	(.135)	(.0049)	(.08)
Stochastic -MC	$ 8.40	$8.52	$.118	1.40
	(.137)	(.134)	(.0043)	(.07)

NOTE- Standard error of stochastic figures from 500 iterations through rainfall realizations. The deterministic scenario assumes average annual rainfall each year. Stochastic assumes i.i.d. random draws from high, average, low rainfall state based on the empirical probabilities. Stochastic- MC assumes draws from an Markov chain process where the transition probabilities are found in TABLE 2.

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Research Questions

- How much does the loss of irrigated land above and aquifer affect the magnitude of theoretical gains from management?
  - 1.24 %
    - Depends on the relative value of "backstop technology"
    - Interest rate
    - Farming intensity (% of area farmed)

- What effect does uncertainty in rainfall and specification of the stochastic process have on gains from management?
    - Increase in gains (.15-.16%), induces slightly larger water savings than under deterministic rules
    - Policy functions differ
    - Markov chain leads to slightly larger welfare gains from management

Implications

- Groundwater Management
  - Scope for gains in welfare with reductions (15%) in extraction rates. Small % gains.
  - Depends on expectations matching progression of climate
  - Variable rules to induce savings in better and average years to have in drought years

- Resource savings for an uncertain future?
  - Non-stationary or uncertain rainfall distributions

REFERENCES

Bellman, R., 1957. *A Markovian decision process*, DTIC Document. Available at: http://oai.dtic.mil/oai/oai?verb=getRecord&metadataPrefix=html&identifier=AD0606367 [Accessed January 24, 2014].

Brito, P., 2007a. Introduction to Dynamic Programming Applied to Economics. Available at: http://www.fep.up.pt/docentes/joao/material/aea/notas_pbrito_2008.pdf [Accessed January 24, 2014].

Brito, P., 2007b. Introduction to Dynamic Programming Applied to Economics. Available at: http://www.fep.up.pt/docentes/joao/material/aea/notas_pbrito_2008.pdf [Accessed May 13, 2014].

C.S. Kim, Michael R. Moore & John Hanchar, 1989. A Dynamic Model of Adaptation to Resource Depletion: Theory and an Application to Groundwater Mining. *Journal of Environmental Economics and Management*, 7(1), pp.66–82.

Gisser, M. & Sanchez, D.A., 1980. Competition versus optimal control in groundwater pumping. *Water Resources Research*, 16(4), pp.638–642.

Guilfoos, T. et al., 2013. Groundwater management: The effect of water flows on welfare gains. *Ecological Economics*, 95, pp.31–40.

Hendricks, N.P. & Peterson, J.M., 2012. Fixed effects estimation of the intensive and extensive margins of irrigation water demand. Available at: http://krex.k-state.edu/dspace/handle/2097/13952 [Accessed January 29, 2014].

Hornbeck, R. & Keskin, P., 2012. *Does Agriculture Generate Local Economic Spillovers? Short-run and Long-run Evidence from the Ogallala Aquifer*, National Bureau of Economic Research. Available at: http://www.nber.org/papers/w18416 [Accessed January 24, 2014].

Knapp, K.C. & Olson, L.J., 1995. The economics of conjunctive groundwater management with stochastic surface supplies. *Journal of Environmental Economics and Management*, 28(3), pp.340–356.

Koundouri, P., 2004. Current issues in the economics of groundwater resource management. *Journal of Economic Surveys*, 18(5), pp.703–740.

McGuire, V.L., 2012. Water-level and storage changes in the High Plains aquifer, predevelopment to 2011 and 2009–11. Available at: http://digitalcommons.unl.edu/usgspubs/123/ [Accessed May 22, 2014].

Putterman, M.L., 1994. Markov decision processes. *John Wiely and Sons, New York*.

Savage, J. & Brozovic, N., 2011. Spatial externalities and strategic behavior in groundwater pumping. In *AERE Summer Conference*. Available at: http://webmeets.com/files/papers/AERE/2011/188/savage_gw_5_21_11.pdf [Accessed May 22, 2014].

Srikanthan, R. & McMahon, T.A., 1999. Stochastic generation of annual, monthly and daily climate data: A review. *Hydrology and Earth System Sciences*, 5(4), pp.653–670.

Steward, D.R. et al., 2013. Tapping unsustainable groundwater stores for agricultural production in the High Plains Aquifer of Kansas, projections to 2110. *Proceedings of the National Academy of Sciences*, 110(37), pp.E3477–E3486.

The Economist, 2013. Water and Agriculture in Kansas: Sip it Slowly. *The Economist*.

Tsur, Y. & Graham-Tomasi, T., 1991. The buffer value of groundwater with stochastic surface water supplies. *Journal of Environmental Economics and Management*, 21(3), pp.201–224.

Methods

Bellman equation

$$V(x_t) = \max_{w_t}\{\, \pi_t(w_t, x_t|r_t) + \beta E_t[V(g(w_t, x_t|r_{t+1})]\}$$

Methods

Bellman equation

Equation of motion

$$V(x_t) = \max_{w_t}\{ \underline{\pi_t(w_t, x_t|r_t)} + \underline{\beta E_t[V(g(w_t, x_t|r_{t+1}))]}\}$$

One year's profit

Present (expected) value of the groundwater stock in the next period to infinity

Methods

Bellman equation

$$V(x_t) = \max_{w_t}\{ \underline{\pi_t(w_t, x_t|r_t)} + \underline{\beta E_t[V(g(w_t, x_t|r_{t+1}))]}\}$$

One year's profit

Present (expected) value of the groundwater stock in the next period to infinity

- V(x) is the value function, or the present value of the system assuming optimal management in all subsequent periods
  - Principle of optimality

Methods

- Iteration:
  - 1: Guess at form of V(x)
  - 2: Maximize Bellman for each (discrete) level of X, call this V'(X)

$$V'(x_t) = \max_{w_t}\{ \pi_t(w_t, x_t|r_t) + \beta E_t[V(g(w_t, x_t|r_{t+1})]\}$$

  - 3: Calculate difference between V'(X) and V(X)
    - Stop if difference is small enough (tolerance)
  - if not : Replace your initial guess of V(X) with maximized V'(X) and start over

- Converges to V(X)

- Recover policy function w(X,r)

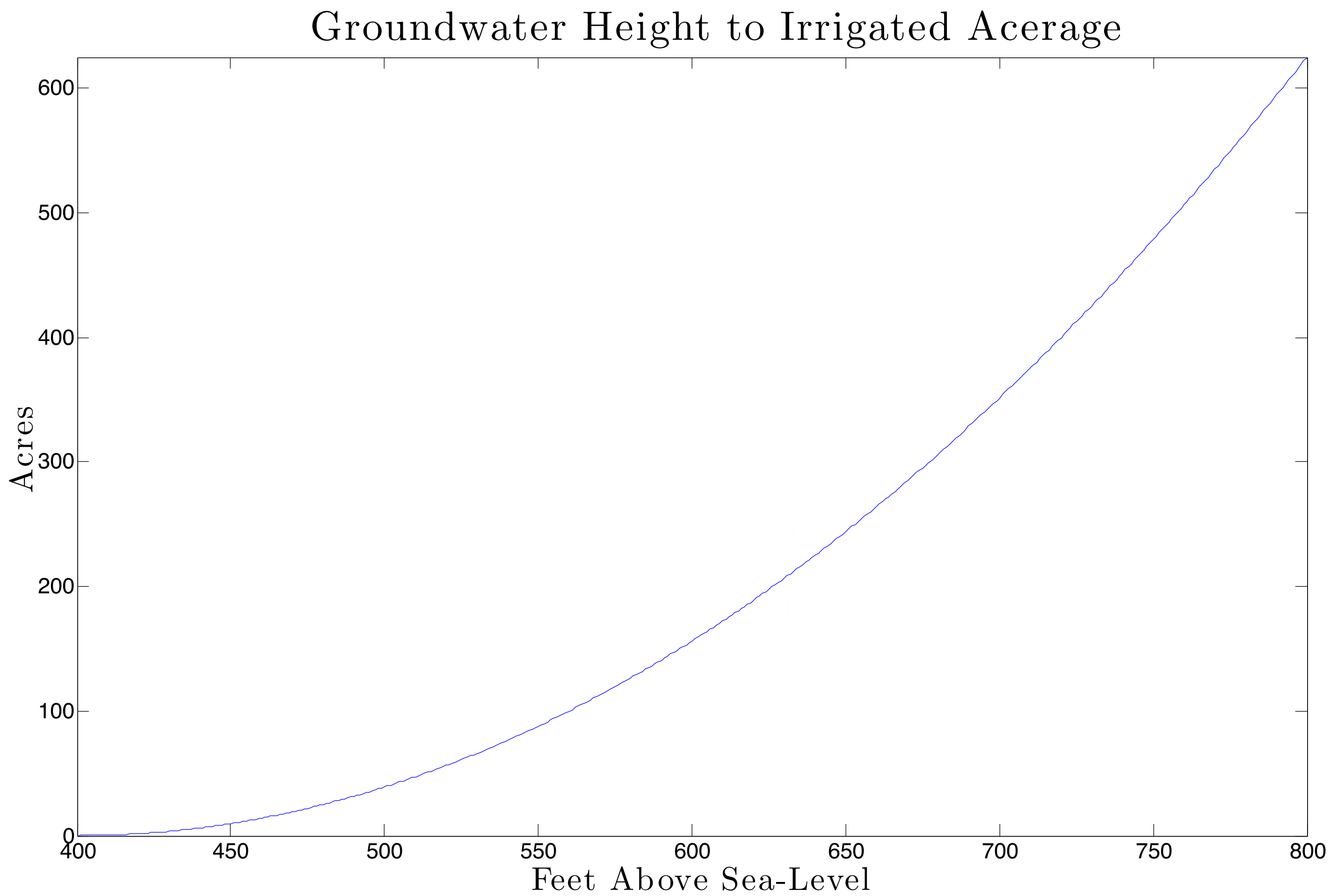
Table 2. Testing the Robustness of GSE

Source	Model	Welfare Gains	Basin/Location	Recharge
		1980–1985		
Gisser and Sanchez [1980a, 1980b]	baseline model	0.01% (r = 10%)	Pecos/New Mexico	negligible
Noel et al. [1980]	baseline model	10.00% (r = 10%)	Yolo/Calofornia	moderate
Lee et al. [1981]	baseline model	0.30% (r = 10%)	Ogallala/Texas	negligible
Feinerman and Knapp [1983]	baseline model	10.00% (r = 5%)	Kern/California	substantial
Allen and Gisser [1984]	nonlinear demand	0.01% (r = 10%)	Pecos/New Mexico	negligible
Nieswiadomy [1985]	baseline model	0.28% (r = 10%)	High Plains/Texas	moderate
Worthington et al. [1985]	variable productivity	28.98% (r = 6%)	Crow Gree/Montana	moderate
		1986 to Today		
Kim et al. [1989]	demand adaptation	1–3.7% (r = 5–2%)	High Plains/Texas	moderate
Dixon [1989]	stochastic DP	0.3% (r = 5%)	Kern/California	substantial
Provencher [1993]	stochastic DP	2–3% (r = 5%)	Madera/California	substantial
Brill and Burness [1994]	demand growth (2% p.a.)	16.85% (r = 1%)	Ogallala/California%	negligible
Provencher and Burt [1994]	stochastic DP	4% (r = 5%)	Kern/California	substantial
Knapp and Olson [1995]	stochastic OC	2.6% (r = 5%)	Kern/California	substantial
Koundouri [2000]	adaptation/near depletion	409.4% (r = 5%)	Kiti/Cyprus	negligible
Burness and Brill [2001]	substitutable technology	2.2% (r = 4%)	Curry/New Mexico%	negligible

Increases in	Effect on Welfare Gains
Sensitivity Analysis	
Aquifer area[a]	negative and moderate
Aquifer storativity[a]	negative and moderate
Surface inflow[a]	positive and small
Initial lifts[a]	negative and small
Energy costs[a]	positive and small
Interest rate[a]	negative and large
Demand intercept[b]	positive and Moderate
Demand slope[b]	positive and large

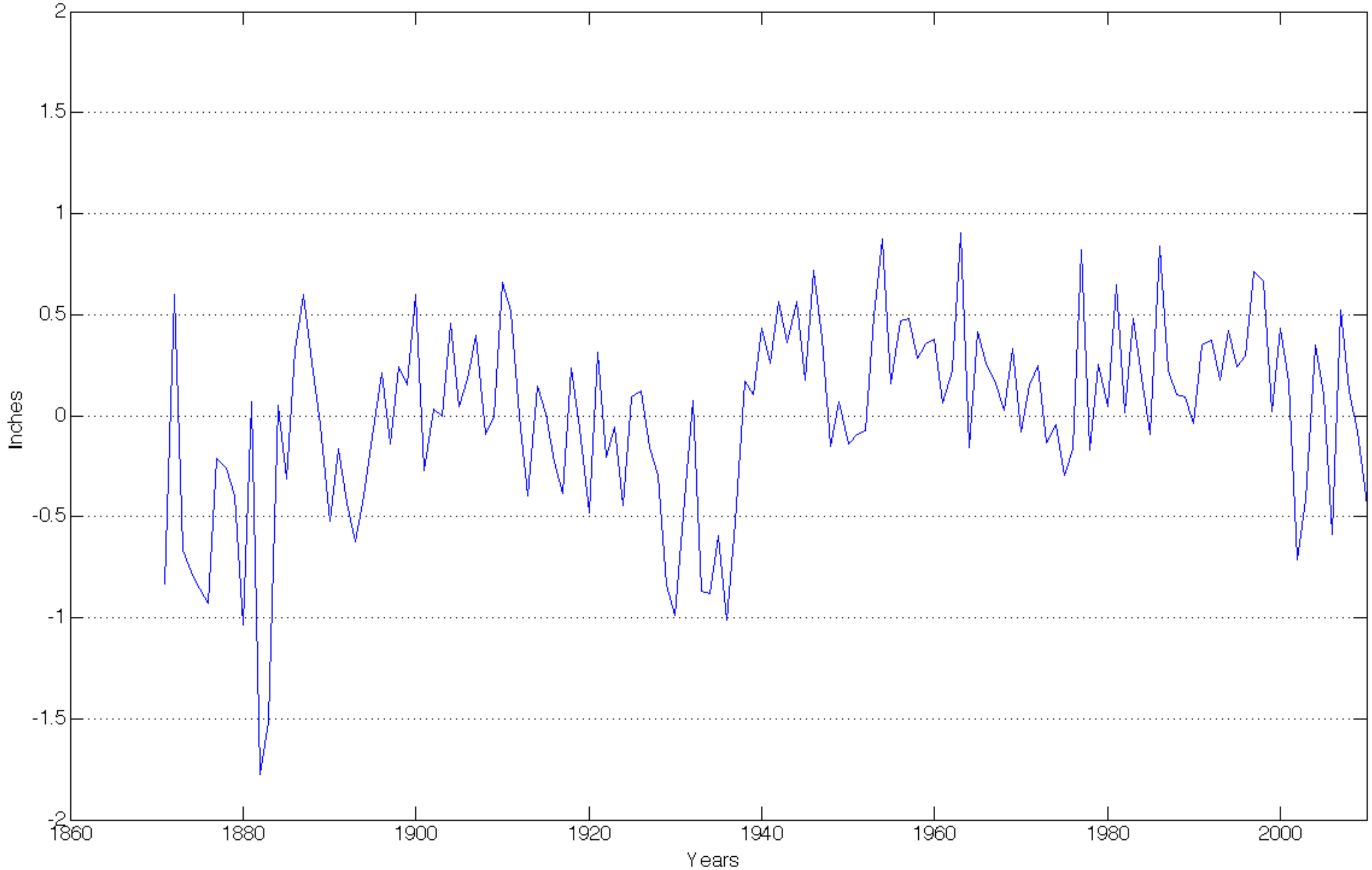
[a]See, for example, *Feinerman and Knapp* [1983].
[b]See, for example, *Nieswiadomy* [1985].

Model



Groundwater Height to Irrigated Acerage
Acres
Feet Above Sea-Level
600
500
400
300
200
100
0
400
450
500
550
600
650
700
750
800

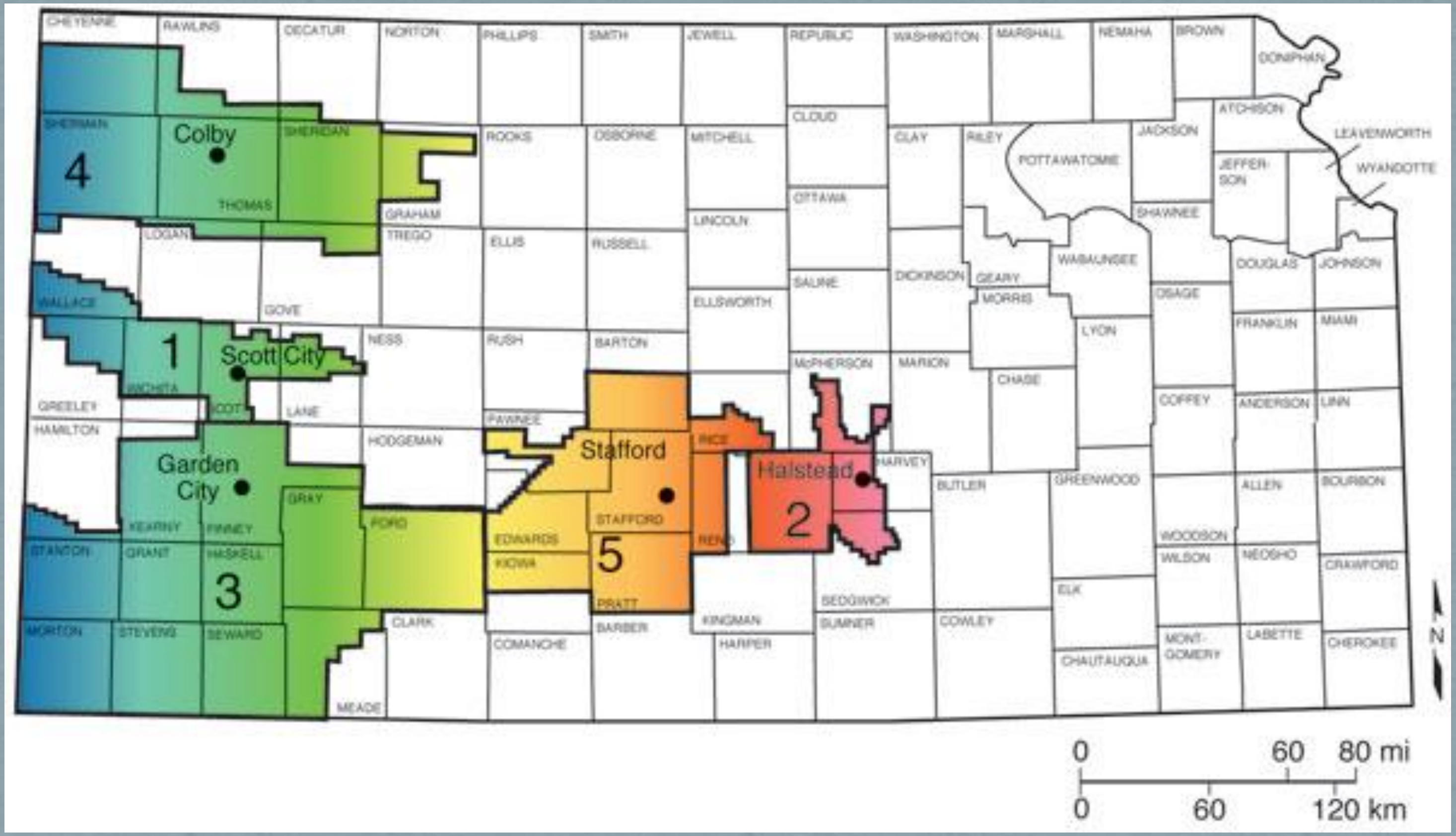
FIGURE 1
Yearly Deviations From Average Annual Rainfall
Northwestern Kansas



NOTE- Rainfall is from NOAA-CIRES Century Reanalysis for years 1871 to 2011 for 1 degree grid centered at 41° N and 259° W. Average annual rainfall over the period is 7.96 inches

Relevant Works

- Bathtub - little gain from optimal management
  - Gisser and Sanchez 1980
  - C.S. Kim et al. 1989 – Endogenous crop switching
  - Tsur and Tomasi 1991 – Uncertainty and buffer values
  - Knapp and Olson 1995 – Uncertainty and possible artificial recharge
- Civil Engineering
  - Steward 2013 - extraction reduction scenarios, heterogeneous exhaustion


Colby
4
1
Scott City
Garden City
3
Stafford
5
Halstead
2
0
60
80 mi
0
60
120 km
N

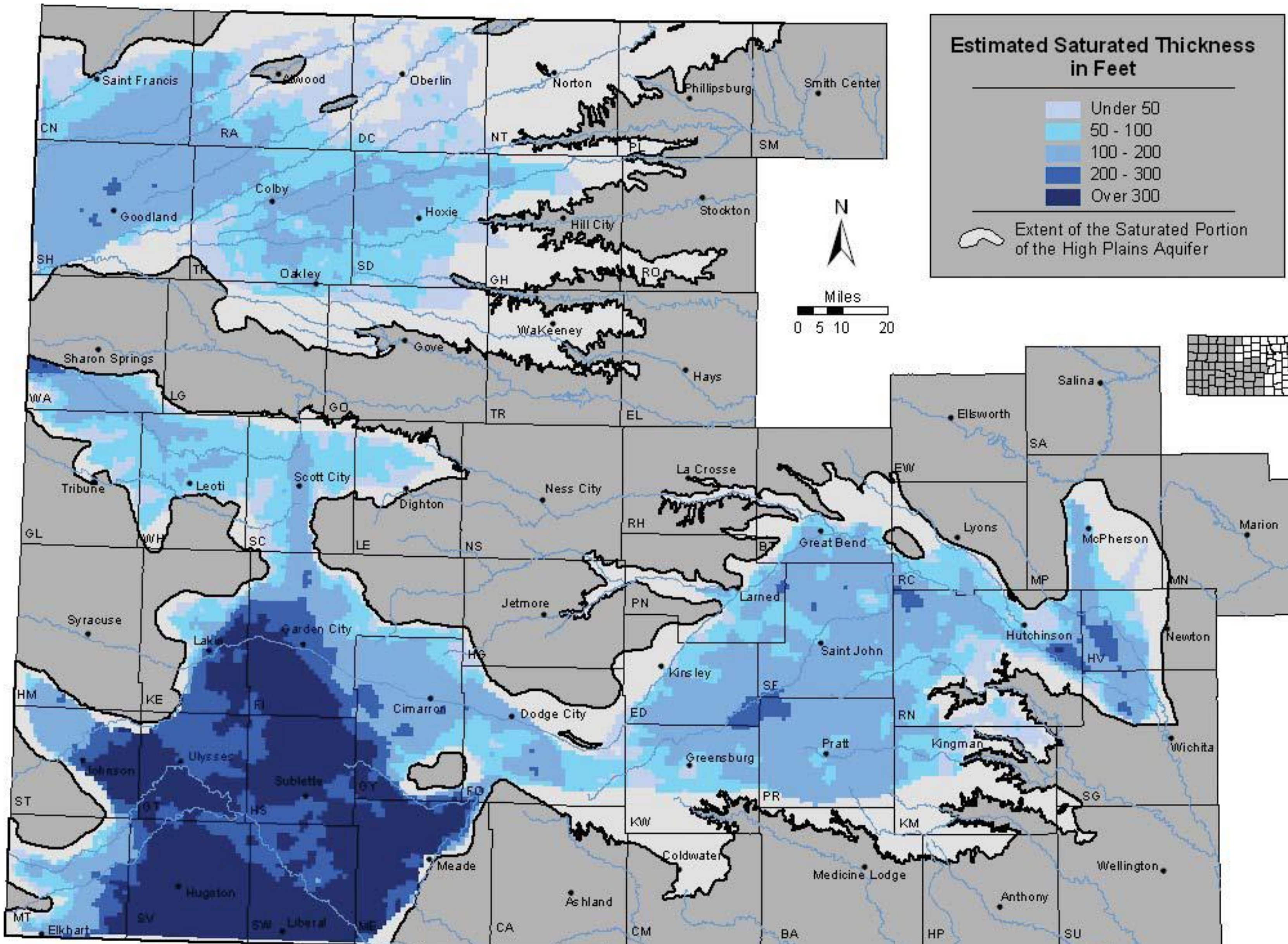
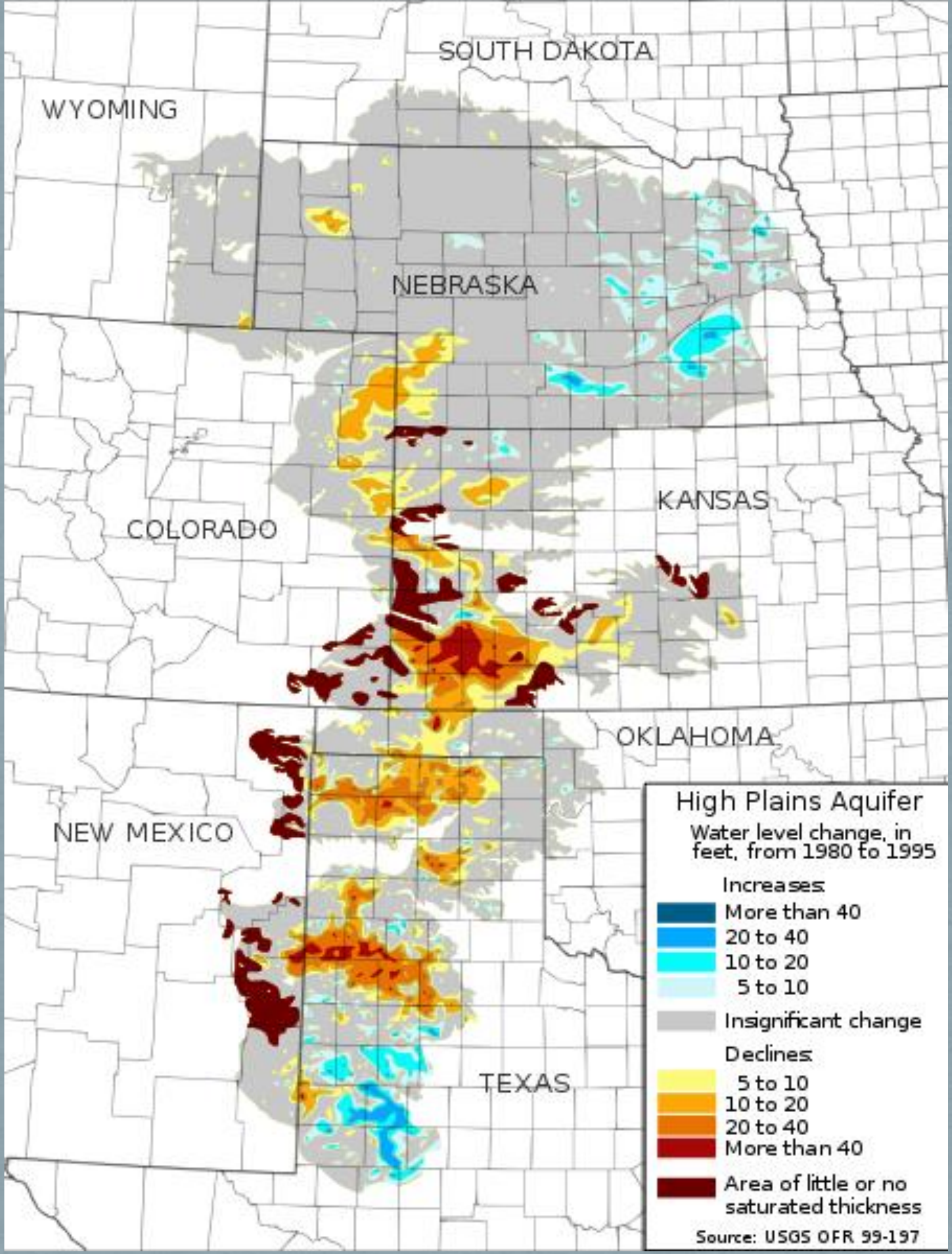


Figure 4—**Predevelopment saturated thickness for the High Plains aquifer in Kansas. Kansas Geological Survey(2009)**


SOUTH DAKOTA
WYOMING
NEBRASKA
KANSAS
COLORADO
OKLAHOMA
NEW MEXICO
TEXAS
High Plains Aquifer
Water level change, in feet, from 1980 to 1995
Increases
More than 40
20 to 40
10 to 20
5 to 10
Insignificant change
Declines
5 to 10
10 to 20
20 to 40
More than 40
Area of little or no saturated thickness
Source: USGS OFR 99-197